

- $\frac{d}{dx} [cf(x)] = cf'(x)$
- $\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$
- $\frac{d}{dx} [x^n] = nx^{n-1}$
- $\frac{d}{dx} [e^{f(x)}] = f'(x)e^{f(x)}$
- $\frac{d}{dx} [a^x] = (\ln a) a^x$
- $\frac{d}{dx} [\log_c x] = \frac{1}{x \ln c}$
- $\frac{d}{dx} [\ln(f(x))] = \frac{f'(x)}{f(x)}$
- $\frac{d}{dx} [\sin x] = \cos x$
- $\frac{d}{dx} [\cos x] = -\sin x$

- $\frac{d}{dx} [\tan x] = \sec^2 x$
- $\frac{d}{dx} [\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} [\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx} [\tan^{-1} x] = \frac{1}{1+x^2}$
- Product rule:  $\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + g'(x)f(x)$
- Quotient rule:  $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$ .
- Chain rule:  $\frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$
- Marginal cost:  $c'(x)$
- Average cost:  $\frac{\Delta c}{\Delta x} = \frac{c(x_2) - c(x_1)}{x_2 - x_1}$

- Area between curves (ch 6.1)

(provided that on the interval  $[a, b]$ ,  $f(u) \geq g(u)$ )

$$A = \int_a^b [f(u) - g(u)] du$$

Essentially,  $f(u) - g(u)$  needs to be {right function} - {left function} if the functions are in terms of  $y$  and you have  $dy$ .

Otherwise,  $f(u) - g(u)$  needs to be {upper function} - {lower function} if the functions are in terms of  $x$  and you have  $dx$ .

- Volume (ch 6.2, 6.3)

– General:

$$V(u) = \int_{u=a}^{u=b} A(u) du$$

– Disk Method

rotation about x-axis:  $V(x) = \int_{x=a}^{x=b} \pi (f(x))^2 dx$

rotation about y-axis:  $V(y) = \int_{y=a}^{y=b} \pi (g(y))^2 dy$

– Cylindrical shells

rotation about y-axis:  $V(x) = \int_{x=a}^{x=b} (2\pi x) (f(x)) dx$

rotation about x-axis:  $V(y) = \int_{y=a}^{y=b} (2\pi y) (g(y)) dy$

- Arc Length (ch 6.4)

– If  $x = f(t)$  and  $y = g(t)$

$$L = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

– If  $x = x$  and  $y = g(x)$

$$L = \int_{x=a}^{x=b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

– If  $x = f(y)$  and  $y = y$

$$L = \int_{y=a}^{y=b} \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy$$

- Average Value (ch 6.5)

$$f_{avg} = \frac{1}{b-a} \int_a^b f(x) dx$$

- Work (ch 6.6)

$$W = \int_a^b f_{orce}(x) dx$$

- Center of Mass (ch 6.6)

– Moments

$$M_y = \rho \int_a^b x f(x) dx$$

$$M_x = \rho \int_a^b \frac{1}{2} [f(x)]^2 dx$$

– Centroid

$$\bar{x} = M_y / \left( \rho \int_a^b f(x) dx \right)$$

$$= \frac{1}{A} \int_a^b x f(x) dx$$

$$\bar{y} = M_x / \left( \rho \int_a^b f(x) dx \right)$$

$$= \frac{1}{A} \int_a^b \frac{1}{2} [f(x)]^2 dx$$

- Economic Surplus (ch 6.7)

– Consumer Surplus

Given a “Production Level” of  $C$ , then  $P = p(C)$  and Consumer Surplus is

$$\int_0^C [p(x) - P] dx$$

– Producer Surplus

Given a “Production Level” of  $C$ , then  $P = p(C)$  and Producer Surplus is

$$\int_0^C [P - p(x)] dx$$

- Probability (ch 6.8)

– Probability Density Function

$f(x)$  is a probability density function if both of the following are true:

\*  $f(x) \geq 0$  for all  $x$

\*  $1 = \int_{-\infty}^{\infty} f(x) dx$

– Probability of an event

If  $f(x)$  is a probability density function, then

$$\mathbb{P}(a \leq X \leq b) = \int_a^b f(x) dx$$

– Average Value (Mean)

if  $f(x)$  is a probability density function, then the mean or average value is given by

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$